Root Loci Based Small-Signal Stabilization of Longitudinal Networks Using Supplementary Stabilizing Signals

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ABSTRACT

A novel scheme for enhanced small-signal stability in power system networks of longitudinal structure is proposed. This is achieved through incorporation of supplementary stabilizing signals derived from measurements at remote generating units using dedicated communication links. Root loci methods are employed to adjust the parameters of the power system stabilizers. The theoretical foundations underlying the concept are discussed and the relative merits investigated.

1. INTRODUCTION

A typical example for a power system of longitudinal structure is found in England. Inter-area oscillations over the tie lines linking the northern and southern parts of the country have been experienced in the past. As a result, it has been necessary to tighten network operating constraints. In this paper it is shown that through the use of communication links the inter-area mode can be damped as effectively as the other electromechanical oscillation modes. In this context the benefits of additional non-local information added to input signals of power system stabilizers (PSS) was investigated.

In Section 2 of this paper, a four-machine two-area power system model, which is representative for a longitudinal power system network, is described and analyzed. The foundations for the use of supplementary stabilizing signals for power system stabilizers are developed in Section 3. In Section 4, the realization of the concept is discussed. In Section 5, robustness and failsafeness considerations are addressed. Conclusions are summarized in Section 6.

2. STUDY SYSTEM

In this section an example of a network of longitudinal structure is described. This network serves as a study system for the upcoming considerations. Using eigenanalysis tools, the properties of the study sytem are investigated.

2.1 Structure

The study system is depicted in Fig. 1. It is of equal structure as the network used in [7]. The study system is of longitudinal structure, two areas are connected through a weak tie line. All generating units are equipped with automatic voltage regulators with potential-source controlled-rectifier exciters [4] and electrohydraulic speed governors [5]. The inertia constants of the generating units within an area are equal but those of area 2 are higher than the inertia constants of area 1. Reactive compensation is provided at load buses 7 and 9 to obtain a satisfactory voltage profile.

The generators are represented by the detailed synchronous machine model based on model 2.2 [10]. Unless stated otherwise, no power flow over the tie line and constant current load characteristics are assumed.

2.2 Eigenanalysis

All eigenvalues of the study system are distinct so that a transformation of the state-space realization to diagonal form is feasible [3]. Since the study system comprises four synchronous generating units, three modes of electromechanical oscillations are observed. In addition, a further low-frequency oscillation mode occurs due to the actions of the speed governing systems.

Eigenvalues and damping ratios [6] of the low-frequency oscillation modes of the study system are shown in Table I. The inter-area mode is the most critical of all these low-frequency oscillation modes: The associated damping ratio is negative and causes the whole system to be unstable.

The participation factor gives the participation of a state variable in a mode [8]. The participation factor magnitudes corresponding to rotor speed perturbations of the low-frequency oscillation modes are displayed in Table II. It can be seen that in the inter-area mode and in the governor mode all rotor speed perturbations have a significant participation. In the area 1 local mode the rotor speed perturbations associated with the generating units in area 2 hardly participate and vice versa. The mode shapes [8] of the low-frequency oscillation modes cor-

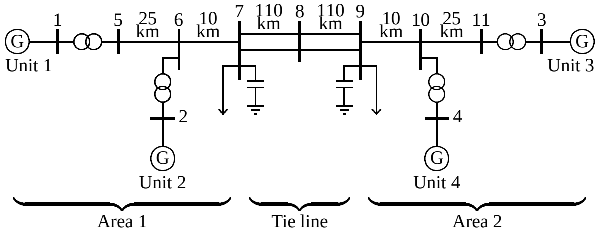

Fig. 1. Study system circuit

Table I. Eigenvalues and damping ratios of study system low-frequency oscillation modes

Mode type	Eigenvalues λ/(rad/s)	Damping ratio ζ
inter-area	0.1170 ± 4.9212j	−0.0238
area 1	−0.3264 ± 8.8734j	0.0368
area 2	−0.3385 ± 7.2277j	0.0468
governor	−0.9301 ± 1.9101j	0.4378

Table II. Participation factor magnitudes corresponding to rotor speed perturbations

State	Participation factor magnitude in mode			
variable	inter-area	area 1	area 2	governor
$\Delta\omega_1$	0.1769	0.2270	0.0021	0.1138
$\Delta\omega_2$	0.1332	0.2738	0.0003	0.1139
$\Delta\omega_3$	0.1156	0.0000	0.2286	0.1950
$\Delta\omega_4$	0.0662	0.0009	0.2834	0.1927

responding to rotor speed perturbations are depicted in Fig. 2. In the inter-area mode the generating units in one area swing coherently and against the generating units located in the other area. In the governor mode all rotor speed perturbations are approximately in phase. In the local modes the generating units within one area swing against each other.

3. RESPONSIVENESS TOWARDS INTER-AREA OSCILLATIONS IN PSS INPUT SIGNALS

In what follows, based on the concept of the stabilizing feedback loop, eigenanalysis tools are used to assess the merits of using different input signals for power system stabilizers. It will be shown that the incorporation of non-local information can be employed to improve the responsiveness of a PSS input signal towards inter-area oscillations.

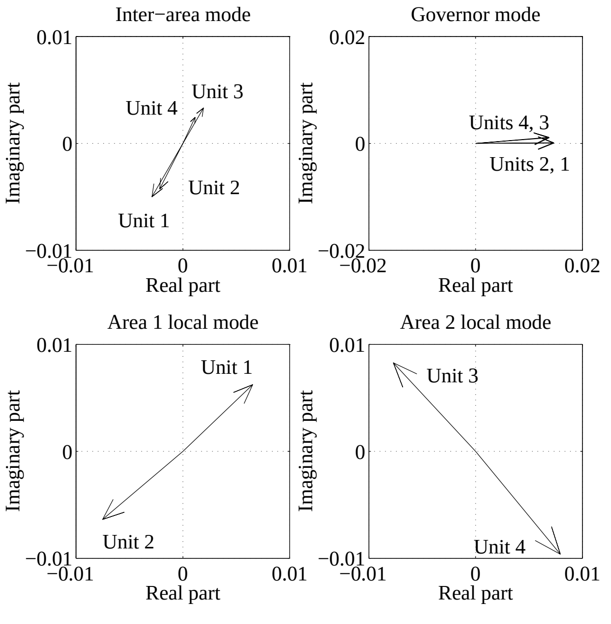

Fig. 2. Mode shapes of low-frequency oscillation modes corresponding to rotor speed perturbations

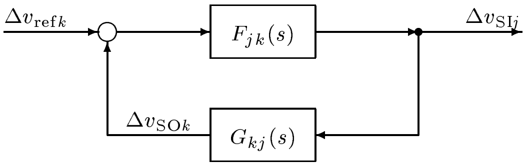

Fig. 3. Stabilizing feedback loop for power systems

3.1 Stabilizing Feedback Loop

A power system stabilizer with transfer function representation $G_{kj}(s)$ inserted in a stabilizing feedback loop is depicted in Fig. 3. Through the PSS output signal $\Delta v_{\mathrm{SO}k}$ the generator excitation is modulated. Transfer function $F_{jk}(s)$ is a single-input single-output (SISO) representation of a generic power system. Input to $F_{jk}(s)$ is the sum of reference voltage perturbation $\Delta v_{\mathrm{ref}k}$ and PSS output signal $\Delta v_{\mathrm{SO}k}$. Output of $F_{jk}(s)$ is signal $\Delta v_{\mathrm{SI}j}$ which is processed by the PSS.

In general, those eigenvalues of the electromechanical oscillation modes which show highest degrees of observability in the output signal of $F_{jk}(s)$ and also highest degrees of controllability through the input signal of $F_{jk}(s)$ can be shifted effectively by the action of $G_{kj}(s)$. An indicator for joint controllability and observability of a mode is the associated residue magnitude of the SISO transfer function $F_{jk}(s)$ [6].

Transfer function residue analysis can therefore be used to determine suitable locations for power system stabilizers [9] and to tune power system stabilizers [1].

3.2 Inter-Area Mode Responsiveness in Local Signals

In the context of the following deliberations, a PSS installation at generating unit 2 is considered. With rotor speed perturbation of unit 2 as PSS input signal, the open loop power system transfer function according to Fig. 3 becomes $F_{22}(s) = \Delta v_{\mathrm{SI2}}/\Delta v_{\mathrm{ref2}}$, where Δv_{SI2} represents the rotor speed perturbation of generating unit 2:

$$\Delta v_{\mathrm{SI2}} = \Delta\omega_2 \quad . \tag{1}$$

The residue magnitudes of $F_{22}(s)$ corresponding to the low-frequency oscillation modes with respect to the residue magnitudes for the eigenvalues of the inter-area mode are shown in Table III. It can be seen that the residue magnitude of the inter-area mode eigenvalues is much lower than the residue magnitude of the area 1 local mode eigenvalues. These results indicate that a PSS which employs $\Delta\omega_2$ as input signal and modulates the exciter input of generating unit 2 is particular effective in shifting the eigenvalues associated with the area 1 local mode.

Table III. Relative transfer function residue magnitudes of $F_{22}(s)$

Transfer function	$\frac{\lvert r_{\mathrm{int}}\rvert}{\lvert r_{\mathrm{int}}\rvert}$	$\frac{\lvert r_{\mathrm{area1}}\rvert}{\lvert r_{\mathrm{int}}\rvert}$	$\frac{\lvert r_{\mathrm{area2}}\rvert}{\lvert r_{\mathrm{int}}\rvert}$	$\frac{\lvert r_{\mathrm{gov}}\rvert}{\lvert r_{\mathrm{int}}\rvert}$
$F_{22}(s)$	1.0000	4.5298	0.0194	0.5872

3.3 Inter-Area Mode Responsiveness in Modified Signals

The data in Table I show that the inter-area mode is the one with the lowest damping ratio. It would therefore be important to find out whether arrangements can be made to add supplementary stabilizing signals in order to improve the responsiveness towards the inter-area mode in the PSS input signal.

In the inter-area mode all generating units of the study system participate, see Table II. This fact suggests to add information about not only the state of the local but of all units which participate in the inter-area mode to the PSS input signal. Given that the rotor speed disturbances are used as state indicators, the following modified PSS input signal is suggested:

$$\Delta v_{\mathrm{SI1}} = \alpha_1 \Delta\omega_1 + \alpha_2 \Delta\omega_2 + \alpha_3 \Delta\omega_3 + \alpha_4 \Delta\omega_4 \quad . \qquad (2)$$

The coefficients $\alpha_k\,,\ k \in \{1\,,\,\dots\,,\,N\}$ are weighting coefficients. Indications on how to select magnitude and sign of these weighting coefficients can be derived from the mode shapes, Fig. 2:

- If only the inter-area mode is excited, all rotor speed disturbances have about the same amplitude. Therefore, for optimal responsiveness towards the inter-area mode in the study system, all weighting coefficients have the same magnitude.
- As typical for networks of longitudinal structure, in the inter-area mode the rotor speed disturbances in one area are in phase but in anti-phase to the rotor speed disturbances of the opposite area. The choice of the sign of the weighting coefficients must reflect this property. The weighting coefficients for rotor speed disturbances from the same area have the same sign while for those corresponding to the other area the opposite sign is chosen.

As a result of these observations, the following weightings lead to a PSS input signal that shows excellent responsiveness towards the inter-area mode:

$$\alpha_1 = 1.0,\ \alpha_2 = 1.0,\ \alpha_3 = -1.0,\ \alpha_4 = -1.0\,. \qquad (3)$$

But while it is important to sufficiently damp the inter-area mode, damping must also be added to the local modes since the corresponding damping ratios are also inadequate, see Table I. An input signal for a PSS which is to be installed in area 1 should therefore be responsive to both inter-area mode and area 1 local mode. By setting the weighting coefficients to

$$\alpha_1 = 1.0,\ \alpha_2 = 1.3,\ \alpha_3 = -1.0,\ \alpha_4 = -1.0\,, \qquad (4)$$

the residue magnitudes of the corresponding transfer function $F_{12}(s) = \Delta v_{\mathrm{SI1}} / \Delta v_{\mathrm{ref2}}$ are as given in Table IV. The weighting coefficient of the local rotor speed perturbation is higher in order to maintain a good responsiveness of Δv_{SI1} towards the area 1 local mode.

The data in Table IV show that the residue of the inter-area mode is set to a higher value compared with the residue of the area 1 local mode. This is desirable for two reasons. Firstly, the inter-area mode is worse damped than the local mode. Secondly, signal Δv_{SI1} represents only a kind of idealized PSS input signal. In reality the rotor speed perturbations are sensed and subjected to various signal processing and transmission procedures before the linear combination of Equation 2 can be established. The total time interval needed for these procedures is expected to be in the range of $50\ \mathrm{ms}$ [2]. This can lead to a reduced responsiveness towards the inter-area mode. To ensure an adequate responsiveness under these conditions, the weighting coefficients of Δv_{SI1} are chosen such that the residue magnitude corresponding to the inter-area mode is considerably higher as the residue magnitude corresponding to the area 1 local mode.

Table IV. Relative transfer function residue magnitudes of $F_{12}(s)$

Transfer function	$\frac{\lvert r_{\mathrm{int}}\rvert}{\lvert r_{\mathrm{int}}\rvert}$	$\frac{\lvert r_{\mathrm{area1}}\rvert}{\lvert r_{\mathrm{int}}\rvert}$	$\frac{\lvert r_{\mathrm{area2}}\rvert}{\lvert r_{\mathrm{int}}\rvert}$	$\frac{\lvert r_{\mathrm{gov}}\rvert}{\lvert r_{\mathrm{int}}\rvert}$
$F_{12}(s)$	1.0000	0.4493	0.0233	0.0049

4. STABILIZATION CONCEPT REALIZATION

In this section, the realization of the proposed PSS concept is discussed. Root loci methods are employed to show that through the use of PSS input signals with enhanced inter-area mode responsiveness the roots corresponding to the inter-area mode can be shifted as effectively as the roots corresponding to the local modes.

4.1 Signal Flow Chart

The following considerations are based on the PSS representation proposed in [4], see Fig. 4. In order to include supplementary signals obtained through measurements taken at $N-1$ remote generating units, additional transducers and gain stages to adjust the weighting coefficients $\alpha_k\,,\ k \in \{1\,,\,\dots\,,\,N\}$, are added, see Fig. 5.

The adherence to the PSS structure in Fig. 4 is advantageous for two reasons. Firstly, it is possible to operate the PSS with supplementary non-local or just with input signals derived from local measurements by adjusting the weighting coefficients accordingly. Secondly, this approach is desirable in order to keep development costs low.

Dedicated telecommunication links are proposed as a suitable medium to transmit signals between the generating units. The specification of these links should be such that the total time interval required for signal processing and transmission does not exceed $50\ \mathrm{ms}$.

4.2 Root Loci

It is initially assumed that there is no power flow over the tie line and the loads show a constant current characteristic. The area 2 local mode is already damped through a PSS installed in area 2.

Following the results of Section 3, the weighting coefficients for the PSS at generating unit 2 are set as follows:

$$\alpha_1 = 1.0,\ \alpha_2 = 1.3,\ \alpha_3 = -1.0,\ \alpha_4 = -1.0\,.$$

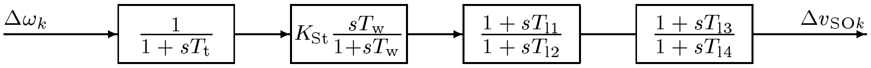

Fig. 4. Power system stabilizer using local input information

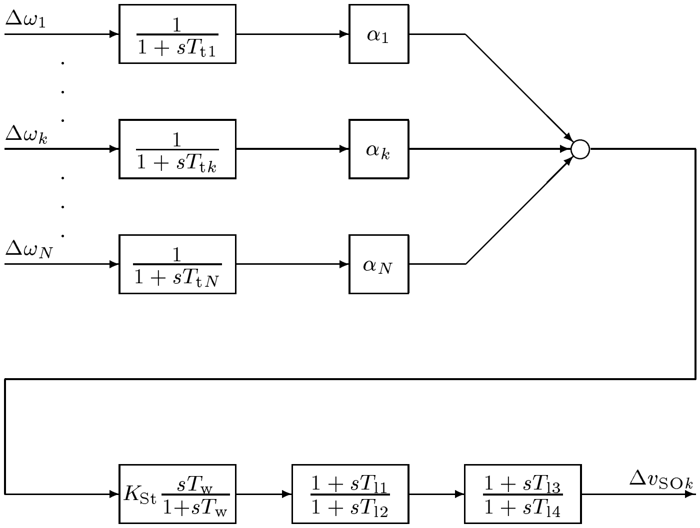

Fig. 5. Power system stabilizer realization using local and supplementary input information

The time constants for the PSS are adjusted as follows:

$$T_{\mathrm{t}1} = 0.05\ \mathrm{s}\,, \quad T_{\mathrm{t}2} = 0.02\ \mathrm{s}\,, \quad T_{\mathrm{t}3} = 0.05\ \mathrm{s}\,,$$
$$T_{\mathrm{t}4} = 0.05\ \mathrm{s}\,, \quad T_{\mathrm{w}} = 15\ \mathrm{s}\,, \quad T_{\mathrm{l}1} = 0.287\ \mathrm{s}\,,$$
$$T_{\mathrm{l}2} = 0.141\ \mathrm{s}\,, \quad T_{\mathrm{l}3} = 0.079\ \mathrm{s}\,, \quad T_{\mathrm{l}4} = 0.161\ \mathrm{s}\,.$$

The time constants $T_{\mathrm{t}k}$ of the transducers for the rotor speed perturbations sensed at remote generating units are given higher values to account for more extensive signal processing and transmission procedures. The value $50\ \mathrm{ms}$ for the first order lag block corresponds to a bandwidth of $20\ \mathrm{rad/s}$.

The time constant T_w of the washout filter, which removes a potential d. c. offset, has been set to a value such that the low-frequency inter-area contributions to the input signal are not reduced.

The phase compensation of the remaining stages has been obtained using root loci techniques. The root loci associated with the low-frequency oscillation modes with variation of the power system stabilizer gain K_St are shown in Fig. 6. It can be recognized that the inter-area mode eigenvalues can be shifted as effectively as the local mode eigenvalues of area 1. The operating point is set at $K_\mathrm{St} = 14.0$. Eigenvalues and damping ratios of the low-frequency oscillation modes for the operating point are summarized in Table V. For all modes in question the damping ratios exceed a value of 0.2 .

Table V. Eigenvalues and damping ratios when unit 2 equipped with PSS using supplementary stabilizing signals

Mode type	Eigenvalues λ/(rad/s)	Damping ratio ζ
inter-area	$-1.4371 \pm 6.6289\mathrm{j}$	0.2119
area 1	$-2.4761 \pm 9.2640\mathrm{j}$	0.2582
area 2	$-2.0989 \pm 7.7566\mathrm{j}$	0.2612
governor	$-1.0542 \pm 1.9193\mathrm{j}$	0.4814

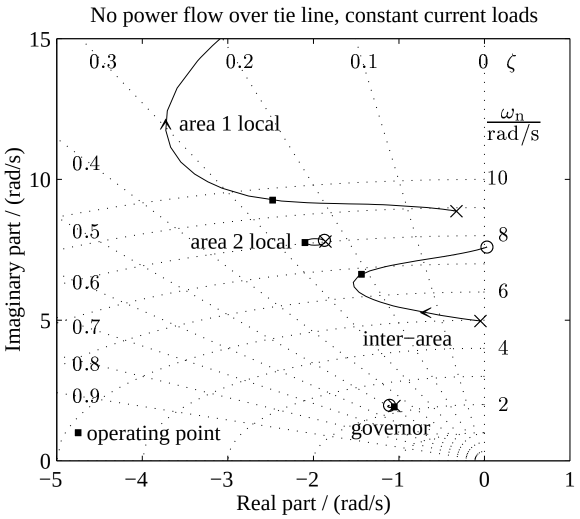

Fig. 6. Root loci when unit 2 equipped with PSS using supplementary stabilizing signals

5. ROBUSTNESS AND FAILSAFENESS

It is of major importance that the design of any stabilization scheme be robust and failsafe. The system must remain stable even under adverse conditions. Different operating modes, changes in network structure and loss of the non-local stabilizing signals are discussed in this context.

5.1 Different Operating Conditions

Two kinds of changes are investigated, changing load characteristics and various levels of power flow over the tie line. In both cases it is assumed that the original PSS parameter settings are retained.

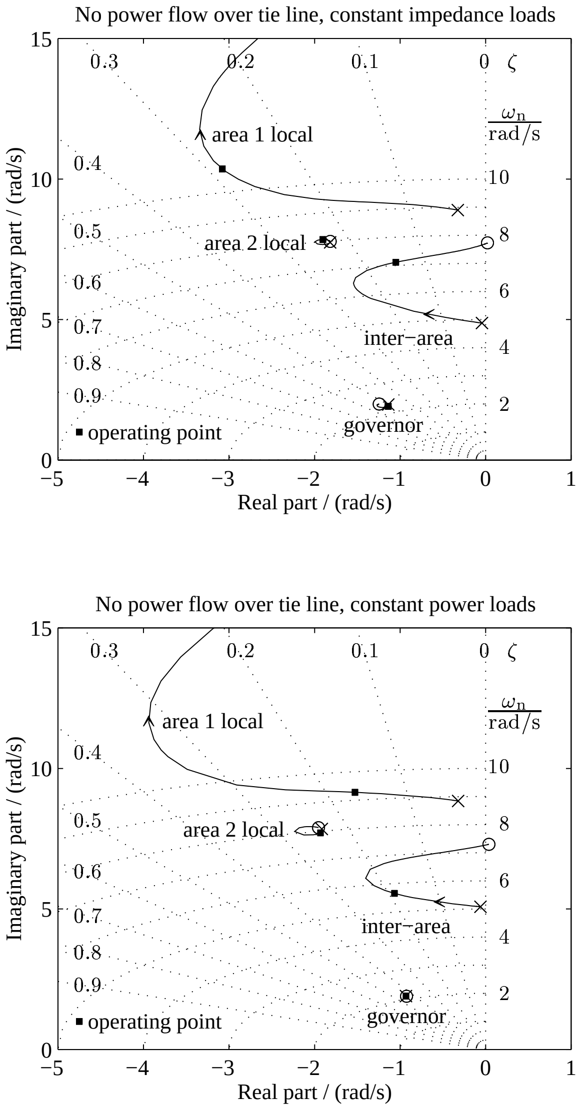

Fig. 7. Root loci for different load characteristics

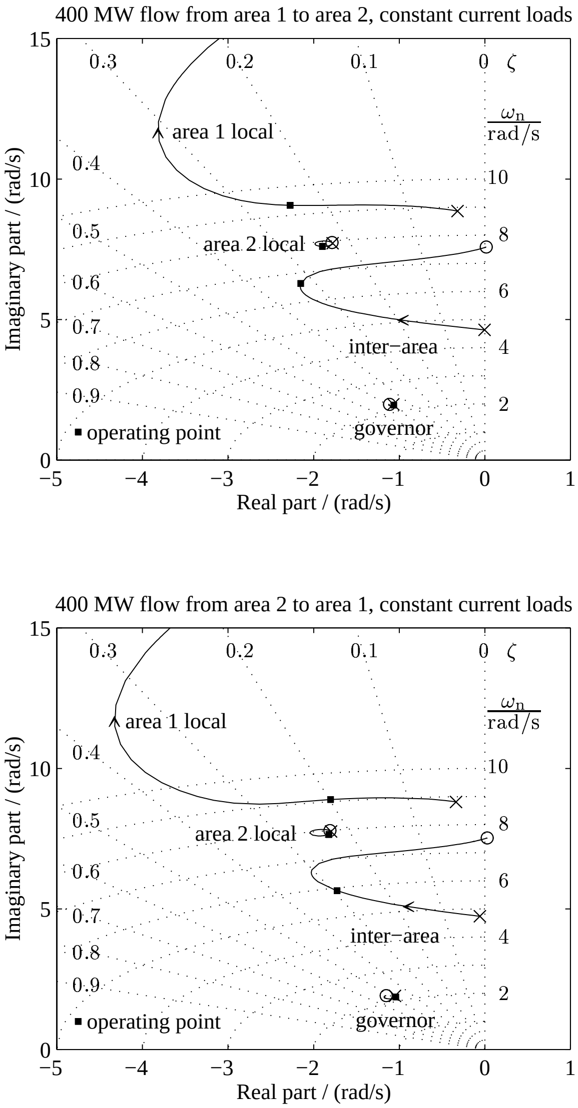

Fig. 8. Root loci for different power flow values over the tie line

In Fig. 7 root loci with variation of K_{St} for different load characteristics are shown. In the first plot the loads adopt a constant impedance characteristic. In the second plot the loads assume a constant power characteristic. The operating point marked in the plots gives the root loci where $K_{\mathrm{St}} = 14.0$, i. e. the value set for constant current loads. Even though the damping of the inter-area mode decreases slightly in both cases for $K_{\mathrm{St}} = 14.0$, the damping of all electromechanical oscillation modes is still very good.

In Fig. 8 root loci with variation of K_{St} for different power flow values over the tie line are given. Constant current loads are assumed. The damping of the inter-area mode remains very good when there is power flow over the tie line in either direction.

The plots demonstrate that the stabilization scheme provides effective inter-area oscillation damping at the operating point, set at $K_{\mathrm{St}} = 14.0$, over the range of practical system conditions considered.

5.2 Changes in Network Structure

Changes in network structure for example occur when it is necessary to take a generating unit out of service for maintenance works. The supplementary stabilizing signals will under such conditions remain beneficial as far as the mode shape of the inter-area mode is largely maintained. Then, the additional transmitted information still contributes to improved responsiveness towards the inter-area oscillation in the PSS input signal.

It can be concluded that the design of the PSS using supplementary stabilizing signals are robust towards changes in network structure as far as the longitudinal character of the network is maintained.

5.3 Loss of Non-Local Information

In the case the supplementary signals are lost, it is important that system stability be maintained.

A time domain test is conducted to assess the significance of the loss of the supplementary information. The

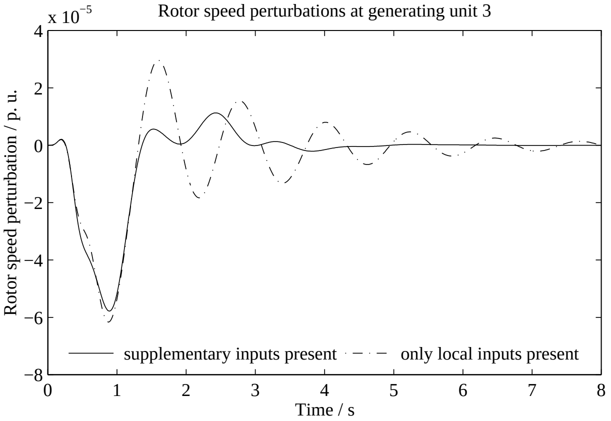

Fig. 9. Rotor speed perturbation responses to $0.01\,\mathrm{p.\,u.}$ exciter impulse input at unit 2, 900 MVA base

case with no initial power flow over the tie line and constant current loads is considered.

A $0.01\,\mathrm{p.\,u.}$ reference voltage impulse at the exciter input of generating unit 2 starting at $t = 0\,\mathrm{s}$ and lasting for $20\,\mathrm{ms}$ is applied. The rotor speed perturbation at generating unit 3 is monitored during a time period of eight seconds.

The simulation results, shown in Fig. 9, are given for the case that supplementary stabilizing signals are present and for the case that the PSS at generating unit 2 processes only locally derived input signals. It is recognizable that the inter-area oscillation decays more rapidly when the supplementary stabilizing signals are present. However, stability is also maintained after loss of the supplementary information.

In the eventuality of a loss of the supplementary signals the damping characteristics can be improved by using retuned PSS parameters. These parameters could be obtained using root-loci design assuming only the availability of locally derived input information. The values of theses parameters would then be prestored so that they are readily available in the case of a contingency.

6. CONCLUSIONS

It was shown that inter-area modes can be damped as effectively as local modes in networks of longitudinal structure. This is achieved by adding supplementary stabilizing signals to input signals of power system stabilizers. Root-loci methods are employed to adjust the parameters of the power system stabilizers. The supplementary information is derived from measurements obtained at remote generating units and then transmitted via communication links.

The performance of the concept was tested in a study performed on a digital computer involving a model of a longitudinal network. It was shown that the stabilization is robust in that it performs well over an appropriate range of practical system conditions. It was demonstrated that a loss of the supplementary signals does not lead to instability and that the ability to return to conventional power system stabilizer operation is supported.

The concept is also an example for the manifold opportunities modern information and communication technology can offer to improve power system performance: The transmission of information over long distances and their subsequent use can improve power system stability.

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